Many industrial applications require power from dc voltage sources. Several of these applications, however, perform better in case these are fed from variable dc voltage sources. Examples of such dc systems are subway cars, trolley buses, battery-operated vehicles, battery-charging etc.

From ac supply systems, variable dc output voltage can be obtained through the use of phase-controlled converters (discussed in Chapter 6) or motor-generator sets. The conversion of fixed dc voltage to an adjustable dc output voltage, through the use of semiconductor devices, can be carried out by the use of two types of dc to dc converters given below [5].

**AC Link Chopper.** In the ac link chopper, dc is first converted to ac by an inverter (dc to ac converter). AC is then stepped-up or stepped-down by a transformer which is then converted back to dc by a diode rectifier Fig. 7.1 (a). As the conversion is in two stages, dc to ac and then ac to dc, ac link chopper is costly, bulky and less efficient.

![Diagram of AC Link Chopper and DC Chopper](image)

(a) AC link chopper (b) dc chopper (or chopper) and
(c) Reproduction of a power semiconductor device.

**Fig. 7.1**

**DC Chopper.** A chopper is a static device that converts fixed dc input voltage to a variable dc output voltage directly Fig. 7.1 (b). A chopper may be thought of as dc equivalent of an ac transformer since they behave in an identical manner. As choppers involve one stage conversion, these are more efficient.

Choppers are now being used all over the world for rapid transit systems. These are also used in trolley cars, marine hoists, forklift trucks and mine haulers. The future electric automobiles are likely to use choppers for their speed control and braking. Chopper systems offer smooth control, high efficiency, fast response and regeneration.
The power semiconductor devices used for a chopper circuit can be power BJT, power MOSFET, GTO or force-commutated thyristor. These devices, in general, can be represented by a switch SW with an arrow as shown in Fig. 7.1 (c). When the switch is off, no current can flow. When the switch is on, current flows in the direction of arrow only. The power semiconductor devices have on-state voltage drops of 0.5 V to 2.5 V across them. For the sake of simplicity, this voltage drop across these devices is neglected.

As stated above, a chopper is dc equivalent to an ac transformer having continuously variable turns ratio. Like a transformer, a chopper can be used to step down or step up the fixed dc input voltage. As step-down dc choppers are more common, a dc chopper, or chopper, in this book would mean a step-down dc chopper unless stated otherwise.

The object of this chapter is to discuss the basic principles of chopper operation and the more common types of chopper configurations using ideal switches.

7.1. PRINCIPLE OF CHOPPER OPERATION

A chopper is a high speed on/off semiconductor switch. It connects source to load and disconnects the load from source at a fast speed. In this manner, a chopped load voltage as shown in Fig. 7.2 (b) is obtained from a constant dc supply of magnitude $V_s$. In Fig. 7.2 (a), chopper is represented by a switch SW inside a dotted rectangle, which may be turned-on or turned-off as desired. For the sake of highlighting the principle of chopper operation, the circuitry used for controlled the on, off periods of this switch is not shown. During the period

Fig. 7.2 (a) Elementary chopper circuit and (b) output voltage and current waveforms.

$T_{on}$, chopper is on and load voltage is equal to source voltage $V_s$. During the interval $T_{off}$, chopper is off, load current flows through the freewheeling diode $FD$. As a result, load terminals are short circuited by $FD$ and load voltage is therefore zero during $T_{off}$. In this manner, a chopped dc voltage is produced at the load terminals. The load current as shown in Fig. 7.2 (b) is continuous. From Fig. 7.2 (b), average load voltage $V_0$ is given by

$$V_0 = \frac{T_{on}}{T_{on} + T_{off}} V_s = \frac{T_{on}}{T} V = \alpha V_s \quad \text{(7.1)}$$

where

$T_{on} =$ on-time; $T_{off} =$ off-time

$T = T_{on} + T_{off} =$ chopping period

$\alpha = \frac{T_{on}}{T} =$ duty cycle

Thus load voltage can be controlled by varying duty cycle $\alpha$. Eq. (7.1) shows that load voltage is independent of load current. Eq. (7.1) can also be written as
\[ V_0 = f \cdot T_{on} \cdot V_s \]  
...(7.2)

where \( f = \frac{1}{T} \) = chopping frequency

7.2. CONTROL STRATEGIES

It is seen from Eq. (7.1) that average value of output voltage \( V_0 \) can be controlled through \( \alpha \) by opening and closing the semiconductor switch periodically. The various control strategies for varying duty cycle \( \alpha \) are as follows:

7.2.1. Constant Frequency System

In this scheme, the on-time \( T_{on} \) is varied but chopping frequency \( f \) (or chopping period \( T \)) is kept constant. Variation of \( T_{on} \) means adjustment of pulse width, as such this scheme is also called pulse-width-modulation scheme. This scheme has also been referred to as time-ratio control (TRC) by some authors.

Fig. 7.3 illustrates the principle of pulse-width modulation. Here chopping period \( T \) is constant. In Fig. 7.3 (a), \( T_{on} = \frac{1}{4} T \) so that \( \alpha = 0.25 \) or \( \alpha = 25\% \). In Fig. 7.3 (b), \( T_{on} = \frac{3}{4} T \) so that \( \alpha = 0.75 \) or 75\%. Ideally \( \alpha \) can be varied from zero to infinity. Therefore output voltage \( V_0 \) can be varied between zero and source voltage \( V_s \).

7.2.2. Variable Frequency System

In this scheme, the chopping frequency \( f \) (or chopping period \( T \)) is varied and either (i) on-time \( T_{on} \) is kept constant or (ii) off-time \( T_{off} \) is kept constant. This method of controlling \( \alpha \) is also called frequency-modulation scheme.

Fig. 7.3. Principle of pulse-width modulation (constant \( T \)).

Fig. 7.4 illustrates the principle of frequency modulation. In Fig. 7.4 (a), \( T_{on} \) is kept constant but \( T \) is varied. In the upper diagram of Fig. 7.4 (a), \( T_{on} = \frac{1}{4} T \) so that \( \alpha = 0.25 \). In the lower diagram of Fig. 7.4 (a), \( T_{on} = \frac{3}{4} T \) so that \( \alpha = 0.75 \). In Fig. 7.4 (b), \( T_{off} \) is kept constant and \( T \) is varied. In the upper diagram of this figure, \( T_{on} = \frac{1}{4} T \) so that \( \alpha = 0.25 \) and in the lower diagram \( T_{on} = \frac{3}{4} T \) so that \( \alpha = 0.75 \).

Frequency modulation scheme has some disadvantages as compared to pulse-width modulation scheme. These are as under:
Fig. 7.4. Principle of frequency modulation.

(a) on-time $T_{on}$ constant and (b) off-time $T_{off}$ constant.

(i) The chopping frequency has to be varied over a wide range for the control of output voltage in frequency modulation. Filter design for such wide frequency variation is, therefore, quite difficult.

(ii) For the control of $\alpha$, frequency variation would be wide. As such, there is a possibility of interference with signalling and telephone lines in frequency modulation scheme.

(iii) The large off-time in frequency modulation scheme may make the load current discontinuous which is undesirable.

It is seen from above that constant frequency (PWM) scheme is better than variable frequency scheme. PWM technique has, however, a limitation. In this technique, $T_{on}$ cannot be reduced to near zero for most of the commutation circuits used in choppers. As such, low range of $\alpha$ control is not possible in PWM. This can, however, be achieved by increasing the chopping period (or decreasing the chopping frequency) of the chopper.

### 7.3. STEP-UP CHOPPERS

For the chopper configuration of Fig. 7.2 (a), average output voltage $V_o$ is less than the input voltage $V_s$, i.e. $V_o < V_s$; this configuration is therefore called step-down chopper. Average output voltage $V_0$ greater than input voltage $V_s$ can, however, be obtained by a chopper called step-up chopper. Fig. 7.5 (a) illustrates an elementary form of a step-up chopper. In this article, working principle of a step-up chopper is presented.
In this chopper, a large inductor \( L \) in series with source voltage \( V_s \) is essential as shown in Fig. 7.5 (a). When the chopper \( CH \) is on, the closed current path is as shown in Fig. 7.5 (b) and inductor stores energy during \( T_{on} \) period. When the chopper \( CH \) is off, as the inductor current cannot die down instantaneously, this current is forced to flow through the diode and load for a time \( T_{off} \). As the current tends to decrease, polarity of the emf induced in \( L \) is reversed as shown in Fig. 7.5 (c). As a result, voltage across the load, given by \( V_0 = V_s + L \cdot di/dt \), exceeds the source voltage \( V_s \). In this manner, the circuit of Fig. 7.5 (d) acts as a step-up chopper and the energy stored in \( L \) is released to the load.

When \( CH \) is on, current through the load would increase from \( I_1 \) to \( I_2 \) as shown in Fig. 7.5 (d). When \( CH \) is off, current would fall from \( I_2 \) to \( I_1 \). With \( CH \) on, source voltage is applied to \( L \) i.e. \( v_L = V_s \). When \( CH \) is off, KCL for Fig. 7.5 (c) gives \( v_L - V_0 + V_s = 0 \), or \( v_L = V_0 - V_s \). Hence \( v_L \) = voltage across \( L \). Assuming linear variation of output current, the energy input to inductor from the source, during the period \( T_{on} \), is

\[
W_{in} = (\text{voltage across } L) \times (\text{average current through } L) \times T_{on} = V_s \cdot \left( \frac{I_1 + I_2}{2} \right) \cdot T_{on} \tag{7.3}
\]

During the time \( T_{off} \), when chopper is off, the energy released by inductor to the load is

\[
W_{off} = (\text{voltage across } L) \times (\text{average current through } L) \times T_{off} = (V_0 - V_s) \cdot \left( \frac{I_1 + I_2}{2} \right) \cdot T_{off} \tag{7.4}
\]

Considering the system to be lossless, these two energies given by Eqs. (7.3) and (7.4) will be equal.

\[
\therefore V_s \left( \frac{I_1 + I_2}{2} \right) \cdot T_{on} = (V_0 - V_s) \cdot \left( \frac{I_1 + I_2}{2} \right) \cdot T_{off}
\]

or

\[
V_0 = V_s \cdot \frac{T}{T_{on}} = V_s \cdot \frac{1}{1 - \alpha} \tag{7.5}
\]
It is seen from Eqn. (7.5) that average voltage across the load can be stepped up by varying the duty cycle. If chopper of Fig. 7.5 (a) is always off, \( \alpha = 0 \) and \( V_0 = V_s \). If this chopper is always on, \( \alpha = 1 \) and \( V_0 = \infty \) (infinity). In practice, chopper is turned on and off so that \( \alpha \) is variable and the required step-up average output voltage, more than source voltage, is obtained.

The principle of step-up chopper can be employed for the regenerative braking of dc motors. In Fig. 7.5 (a), if \( V_s \) represents the motor armature voltage and \( V_0 \) the dc source voltage, the power can be fed back to the dc source in case \( V_s/(1 - \alpha) \) is more than \( V_0 \). In this manner, regenerative braking of dc motor occurs. Even at decreasing motor speeds, regenerative braking can be made to take place provided duty cycle \( \alpha \) is so adjusted that \( V_s/(1 - \alpha) \) exceeds the fixed source voltage \( V_0 \).

**Example 7.1** For the basic dc to dc converter of Fig. 7.2 (a), express the following variables as functions of \( V_s \), \( R \) and duty cycle \( \alpha \) in case load is resistive:

(a) Average output voltage and current
(b) Output current at the instant of commutation
(c) Average and rms freewheeling diode currents
(d) Rms value of the output voltage
(e) Rms and average thyristor currents
(f) Effective input resistance of the chopper.

**Solution.** The load voltage variation is shown in Fig. 7.2 (b). For a resistive load, output or load current waveform is similar to load voltage waveform.

(a) Average output voltage, \( V_0 = \frac{T_{on}}{T} V_s = \alpha V_s \)

Average output current,
\[
I_0 = \frac{V_0}{R} = \frac{T_{on}}{T} \cdot \frac{V_s}{R} = \alpha \frac{V_s}{R}
\]

(b) The output current is commutated by the thyristor at the instant \( t = T_{on} \). Therefore, output current at the instant of commutation is \( V_s/R \).

(c) For a resistive load, freewheeling diode FD does not come into play. Therefore, average and rms values of freewheeling diode currents are zero.

(d) Rms value of output voltage
\[
\left[ \frac{T_{on}}{T} \cdot \frac{V_s^2}{R} \right]^{1/2} = \sqrt{\alpha} \cdot \frac{V_s}{R}
\]

(e) Average thyristor current
\[
\frac{T_{on}}{T} \cdot \frac{V_s}{R} = \alpha \frac{V_s}{R}
\]

Rms thyristor current
\[
\left[ \frac{T_{on}}{T} \cdot \left( \frac{V_s}{R} \right)^2 \right]^{1/2} = \sqrt{\alpha} \cdot \frac{V_s}{R}
\]

(f) Average source current = average thyristor current = \( \alpha \cdot \frac{V_s}{R} \)

Effective input resistance of the chopper
\[
= \frac{\text{dc source voltage}}{\text{average source current}} = \frac{V_s \cdot R}{\alpha \cdot V_s} = \frac{R}{\alpha}
\]
Example 7.2. For type-A chopper of Fig. 7.2 (a), dc source voltage = 230 V, load resistance = 10 Ω. Take a voltage drop of 2 V across chopper when it is on. For a duty cycle of 0.4, calculate
(a) average and rms values of output voltage and
(b) chopper efficiency.

Solution. (a) When chopper is on, output voltage is \((V_s - 2)\) volts and during the time chopper is off, output voltage is zero.

\[
\text{Average output voltage} = \frac{(V_s - 2) \cdot T_{on}}{T} = \alpha (V_s - 2)
\]

\[
= 0.4 \cdot (230 - 2) = 91.2 \text{ V}
\]

Rms value of output voltage,

\[
V_{or} = \left[ (V_s - 2)^2 \cdot \frac{T_{on}}{T} \right]^{1/2} = \sqrt{\alpha} (V_s - 2)
\]

\[
= \sqrt{0.4} \cdot (230 - 2) = 144.2 \text{ V}
\]

(b) Power output or power delivered to load,

\[
P_o = \frac{V_{or}^2}{R} = \frac{(144.2)^2}{10} = 2079.364 \text{ W}
\]

Power input to chopper,

\[
P_i = V_s \cdot I_0 = 230 \cdot \frac{91.2}{10} = 2097.6 \text{ W}
\]

Chopper efficiency

\[
\frac{P_o}{P_i} = \frac{2079.364}{2097.6} \cdot 100 = 99.13\%
\]

Example 7.3. A step-up chopper has input voltage of 220 V and output voltage of 660 V. If the non-conducting time of thyristor-chopper is 100 μs, compute the pulse width of output voltage.

In case pulse width is halved for constant frequency operation, find the new output voltage.

Solution. From Eq. (7.5),

\[
660 = 220 \frac{1}{1 - \alpha}
\]

or

\[
\alpha = 1 - \frac{T_{on}}{T}
\]

\[
\therefore \quad T_{on} = \frac{2}{3} \cdot T \quad \text{and} \quad T_{off} = T - T_{on} = \frac{1}{3} \cdot T = 100 \mu s \quad \text{(given)}
\]

\[
\therefore \quad T_{off} = 300 \mu s \quad \text{and} \quad T_{on} = \frac{2}{3} \cdot 300 = 200 \mu s.
\]

When pulse width is halved, \(T_{on} = \frac{1}{2} \cdot 200 = 100 \mu s\)

for constant frequency operation, \(T = 300 \mu s\); \(T_{off} = T - T_{on} = 200 \mu s\)

\[
\alpha = \frac{T_{on}}{T} = \frac{100}{300} = \frac{1}{3}
\]

\[
\therefore \quad \text{New output voltage, } V_0 = 220 \cdot \frac{1}{1 - \frac{1}{3}} = 330 \text{ V}.
\]

7.4. TYPES OF CHOPPER CIRCUITS

Power semiconductor devices used in chopper circuits are unidirectional devices, i.e., the polarities of output voltage \(V_0\) and the direction of output current \(I_0\) are, therefore, restricted...
A chopper can, however, operate in any of the four quadrants by an appropriate arrangement of semiconductor devices. This characteristic of their operation in any of the four quadrants forms the basis of their classification as type-A chopper, type-B chopper etc. Some authors describe this chopper classification as class A, class B, ... in place of type-A, type-B ..., respectively.

In the chopper-circuit configurations drawn henceforth, the current directions and voltage polarities marked in the power circuit would be treated as positive. In case current directions and voltage polarities turn out to be opposite to those shown in the circuit, these currents and voltages must be treated as negative.

In this section, the classification of various chopper configurations is discussed.

### 7.4.1. First-quadrant, or Type-A, Chopper

This type of chopper is shown in Fig. 7.6 (a). It is observed that chopper circuit of Fig. 7.2 (a) is also type-A chopper. In Fig. 7.6 (a), when chopper CH1 is on, \( v_i = V_s \) and current \( i_o \) flows in the arrow direction shown. When CH1 is off, \( v_i = 0 \) but \( i_o \) in the load continues flowing in the same direction through freewheeling diode FD, Fig. 7.2 (b). It is thus seen that average values of both load voltage and current, i.e. \( V_0 \) and \( i_o \) are always positive: this fact is shown by the hatched area in the first quadrant of \( V_0 - I_o \) plane in Fig. 7.6 (b).

![Fig. 7.6. First-quadrant, or type-A chopper.](image)

The power flow in type-A chopper is always from source to load. This chopper is also called *step-down chopper* as average output voltage \( V_0 \) is always less than the input dc voltage \( V_s \).

### 7.4.2. Second-quadrant, or Type-B, Chopper

Power circuit for this type of chopper is shown in Fig. 7.7 (a). Note that load must contain a dc source \( E \), like a battery (or a dc motor) in this chopper.

![Fig. 7.7. Second-quadrant, or type-B, chopper.](image)
When CH2 is on, \( v_0 = 0 \) but load voltage \( E \) drives current through \( L \) and CH2. Inductance \( L \) stores energy during \( T_{on} \) (= on period) of CH2. When CH2 is off, \( v_0 = \left( E + L \frac{di}{dt} \right) \) exceed source voltage \( V_s \). As a result, diode D2 is forward biased and begins conduction, thus allowing power to flow to the source. Chopper CH2 may be on or off, current \( I_0 \) flows out of the load current \( i_0 \) is therefore treated as negative. Since \( V_0 \) is always positive and \( I_0 \) is negative power flow is always from load to source. As load voltage \( V_0 = \left( E + L \frac{di}{dt} \right) \) is more than source voltage \( V_s \), type-B chopper is also called step-up chopper.

Both type-A and type-B chopper configurations have a common negative terminal between their input and output circuits.

### 7.4.3. Two-quadrant type-A Chopper, or Type-C Chopper

This type of chopper is obtained by connecting type-A and type-B choppers in parallel as shown in Fig. 7.8 (a). The output voltage \( V_0 \) is always positive because of the presence of freewheeling diode FD across the load. When chopper CH2 is on, or freewheeling diode FD conducts, output voltage \( v_0 = V_s \) and in case chopper CH1 is on or diode D2 conducts, output voltage \( v_0 = V_s \). The load current \( i_0 \) can, however, reverse its direction. Current \( I_0 \) flows in the arrow direction marked in Fig. 7.8 (a), i.e. load current is positive when CH1 is on or FD conducts. Load current is negative if CH2 is on or D2 conducts. In other words, CH1 and FD operate together as type-A chopper in first quadrant. Likewise, CH2 and D2 operate together as type-B chopper in second quadrant.

![Fig. 7.8. Two-quadrant type-A chopper, or type-C chopper.](image)

Average load voltage is always positive but average load current may be positive or negative as explained above. Therefore, power flow may be from source to load (first-quadrant operation) or from load to source (second-quadrant operation). Choppers CH1 and CH2 should not be on simultaneously as this would lead to a direct short circuit on the supply lines. This type of chopper configuration is used for motorizing and regenerative braking of dc motors. The operating region of this type of chopper is shown in Fig. 7.8 (b) by hatched area in first and second quadrants.

### 7.4.4. Two-quadrant Type-B Chopper, or Type-D Chopper

The power circuit diagram for two-quadrant type-B chopper, or type-D chopper, is shown in Fig. 7.9 (a). The output voltage \( v_0 = V_s \) when both CH1 and CH2 are on and \( v_0 = -V_s \) when both choppers are off but both diodes D1 and D2 conduct. Average output voltage \( V_0 \) is positive when choppers turn-on time \( T_{on} \) is more than their turn-off time \( T_{off} \) as shown in Fig. 7.9 (a). Average output voltage \( V_0 \) is negative when their \( T_{on} < T_{off} \), Fig. 7.9 (d). The direction of load current is always positive because choppers and diodes can conduct current only in the
direction of arrows shown in Fig. 7.9 (a). As \( V_0 \) is reversible, power flow is reversible. The operation of this type of chopper is shown by the hatched area in first and fourth quadrants in Fig. 7.9 (b).

![Diagram](https://via.placeholder.com/150)

**Fig. 7.9 (a) and (b) Two-quadrant type-B chopper, or type-D chopper (c) \( V_0 \) is positive, \( T_{on} > T_{off} \) and (d) \( V_0 \) is negative, \( T_{on} < T_{off} \)**

**7.4.5. Four-quadrant Chopper, or Type-E Chopper**

The power circuit diagram for a four-quadrant chopper is shown in Fig. 7.10 (a). It consists of four semiconductor switches CH1 to CH4 and four diodes D1 to D4 in antiparallel. Working of this chopper in the four quadrants is explained as under:

**First quadrant**: For first-quadrant operation of Fig. 7.10 (a), CH4 is kept on, CH3 is kept off and \( CH1 \) is operated. With CH1, CH4 on, load voltage \( v_0 = V_s \) (source voltage) and load current \( i_0 \) begins to flow. Here both \( v_0 \) and \( i_0 \) are positive giving first quadrant operation. When CH1 is turned off, positive current freewheels through CH4, D2. In this manner, both \( V_0, i_0 \) can be controlled in the first quadrant.

**Second quadrant**: Here CH2 is operated and CH1, CH3 and CH4 are kept off. With CH2 on, reverse (or negative) current flows through \( L \), CH2, D4 and \( E \). Inductance \( L \) stores energy during the time CH2 is on. When CH2 is turned off, current is fed back to source through diodes D1, D4. Note that here \( E + L \frac{di}{dt} \) is more than the source voltage \( V_s \). As load voltage \( V_0 \) is positive and \( I_0 \) is negative, it is second quadrant operation of chopper. Also, power is fed back from load to source.

**Third quadrant**: For third-quadrant operation of Fig. 7.10 (a), CH1 is kept off, CH2 is kept on and \( CH3 \) is operated. Polarity of load emf \( E \) must be reversed for this quadrant working. With CH3 on, load gets connected to source \( V_s \) so that both \( v_0, i_0 \) are negative leading to third quadrant operation. When CH3 is turned off, negative current freewheels through CH2, D4. In this manner, \( v_0 \) and \( i_0 \) can be controlled in the third quadrant.

**Fourth quadrant**: Here CH4 is operated and other devices are kept off. Load emf \( E \) must have its polarity reversed to that shown in Fig. 7.10 (a) for operation in the fourth quadrant.
quadrant. With CH4 on, positive current flows through CH4, D2, L and E. Inductance L stores energy during the time CH4 is on. When CH4 is turned off, current is fed back to source through diodes D2, D3. Here load voltage is negative, but load current is positive leading to the chopper operation in the fourth quadrant. Also power is fed back from load to source.

The devices conducting in the four quadrants are indicated in Fig. 7.10 (b).

**Example 7.4.** Show that for a basic dc to dc converter, the critical inductance of the filter circuit is given by

$$L = \frac{V_o^2 (V_s - V_o)}{2f V_s P_o}$$

where $V_o$, $V_s$, $P_o$ and $f$ are load voltage, source voltage, load power and chopping frequency respectively.

**Solution.** The critical inductance $L$ is that value of inductance for which the output current falls to zero at $t = T$ during the turn-off period of the chopper. A typical waveform of output current, with critical inductance in the load circuit, is shown in Fig. 7.11 (b). If current variation, from zero to $I_{mx}$ during $T_{on}$ and from $I_{mx}$ to zero during $T_{off}$ is assumed linear, then average value of output current $I_0$ is given by

$$I_0 T = \frac{1}{2} I_{mx} T_{on} + \frac{1}{2} I_{mx} T_{off} = \frac{1}{2} I_{mx} (T_{on} + T_{off}) = \frac{1}{2} I_{mx} T$$

or $I_{mx} = 2 I_0$ = maximum value of chopper current at $t = T_{on}$. It is seen from Fig. 7.11 (a) that when chopper CH is on,
\[
V_0 + L \frac{di}{dt} = V_s \quad \text{or} \quad V_0 + E \frac{I_{mn}}{T_{on}} = V_s
\]

or

\[
L \frac{2I_0}{T_{on}} = V_s - V_0
\]

\[
L = \frac{(V_s - V_0)}{2I_0} T_{on}
\]

But average value of output voltage \( V_0 = f \cdot T_{on} \cdot V_s \) and output, or load, power \( P_0 = V_0 I_0 \). This gives

\[
T_{on} = \frac{V_0}{f \cdot V_s} \quad \text{and} \quad I_0 = \frac{P_0}{V_0}
\]

Substituting these values of \( T_{on} \) and \( I_0 \) in Eq (i), we get

\[
L = \frac{(V_s - V_0) \cdot V_0^2}{2fV_s P_0}
\]

7.5. STEADY STATE TIME-DOMAIN ANALYSIS OF TYPE-A CHOPPER

For the type-A chopper of Fig. 7.6 (a) with RLE load, the waveforms for gate signal \( i_g \), load current \( i_0 \) and load voltage \( v_0 \) are as shown in Fig. 7.12 (a) for continuous conduction and, in Fig. 7.12 (b) for discontinuous conduction. In Fig. 7.12 (b), periodic time T is more than that in Fig. 7.12 (a). The determination of load current expression is useful for knowing (i) the current profile over periodic time T, (ii) the current ripple and (iii) whether the current is continuous or discontinuous. The object of this article is to study the type-A chopper with RLE load for current variation over T, current ripple and also for the Fourier analysis of output voltage.

For RLE type load, \( E \) is the load voltage which may be a dc motor or a battery. When CH1 is on in Fig. 7.6 (a), the equivalent circuit is as shown in Fig. 7.12 (c). For this mode of operation, the differential equation governing its performance is

\[
V_s = R i + L \frac{di}{dt} + E \quad \text{for} \quad 0 \leq t \leq T_{on}.
\]

When CH1 is off, the load current continues flowing through the freewheeling diode and the equivalent circuit is as shown in Fig. 7.12 (d). For this circuit, the differential equation is

\[
0 = R i + L \frac{di}{dt} + E \quad \text{for} \quad T_{on} < t \leq T.
\]

Solution of Eqs. (7.6) and (7.7) may be obtained by the use of Laplace transform. It is seen from Fig. 7.12 (a) that initial value of current is \( I_{mn} \) for Eq. (7.6) and \( I_{mx} \) for Eq. (7.7). Therefore, Laplace transform of Eqs. (7.6) and (7.7) is

\[
RI(s) + L[sI(s) - I_{mn}] = \frac{V_s - E}{s}\]

and

\[
RI(s) + L[sI(s) - I_{mx}] = \frac{E}{s}\]

From Eq. (7.8),

\[
I(s) = \frac{V_s - E}{s(R + L)} + \frac{L \cdot I_{mn}}{R + L} = \frac{V_s - E}{s} + \frac{I_{mn}}{s + \frac{R}{L}}
\]
7.7. MULTIPHASE CHOPPERS

A multiphase chopper is one that consists of two or more choppers connected in parallel. The two-chopper configuration shown in Fig. 7.31 is called a two-phase chopper. Similar three choppers connected in parallel will constitute a 3-phase chopper.

A multiphase chopper may be operated in two modes, viz. in-phase operation mode and the phase-shifted operation mode. In the in-phase operation mode, all the parallel connected choppers are on and off at the same instant. In the phase-shifted operation mode, different choppers are on and off at different instants of time.

In the two-phase chopper configuration shown in Fig. 7.31, inductance L in series with each chopper is assumed to be sufficiently large in that each chopper operates independent of each other. Let the load current be \( I_0 \) and ripple free. For a duty cycle of \( \alpha = 0.30 \), Fig. 7.32 (a) shows the in-phase operation of this chopper when both the choppers are on and off at the same instant.

![Fig. 7.31. Two-phase chopper.](image)

![Fig. 7.32. Input current waveforms for duty cycle \( \alpha = 0.30 \) for (a) in-phase operation and (b) phase-shifted operation.](image)

![Fig. 7.32. Current waveform for phase-shifted operation for (c) \( \alpha = 0.50 \) and (d) \( \alpha = 0.60 \).](image)
The input current $i$, obtained by the addition of $i_1$ and $i_2$, is seen to be doubled as shown. The in-phase operation of multiphase chopper is equivalent to a single-chopper operation.

Fig. 7.32 (b) shows the phase shifted operation for $\alpha = 30\%$. Chopper CH1 is on for $0.3$ T from $t = 0$. Chopper CH2 is made on such that input current obtained from $i_1 + i_2$ is periodic in nature. A comparison of Figs. 7.32 (a) and (b) reveals that for phase-shifted operation, the frequency of input current is doubled and its ripple current amplitude (proportional to $I_{\text{max}} - I_{\text{min}}$) is halved as compared to the inphase-operation of chopper. In the in-phase operating mode, Fig. 7.32 (a) shows that frequency of harmonics in the input current is equal to the switching frequency ($= 1/T$) of each chopper. But in phase-shifted operating mode, Fig. 7.32 (b) shows that frequency of harmonics in the input current is twice the switching frequency ($= 1/T$) of each chopper. As the frequency of harmonics in the input current is twice the switching frequency, the size of filter is reduced in the phase-shifted chopper. This shows that phase-shifted operation of multiphase choppers is usually preferred.

For $\alpha = 50\%$, the input supply current of phase-shifted operation is continuous and without any ripples, Fig. 7.32 (c). For $\alpha = 60\%$, the supply current is continuous but with a pedestal of half the load current, Fig. 7.32 (d).

A multiphase chopper is used where large load current is required. The main advantage of this chopper over a single chopper is that its input current has reduced ripple amplitude and increased ripple frequency. As a consequence of it, size of filter for a multiphase chopper is reduced.

The disadvantages of a multiphase chopper are (i) extra commutation circuits (ii) additional external inductors and (iii) complexity in the control logic.

**Example 7.19.** A type-A chopper operating at $2$ kHz from a $100$ V dc source has a load time constant of $6$ ms and load resistance of $10$ Ω. Find the mean load current and the magnitude of current ripple for a mean load voltage of $50$ V. Also, calculate the minimum and maximum values of load current.

**Solution.** Load time constant, $\frac{L}{R} = 6 \times 10^{-3} \text{ s}$ ; Load resistance, $R = 10$ Ω.

:. Load inductance, $L = 6 \times 10^{-3} \times 10 = 60$ mH

Chopping period, $T = \frac{1}{f} = \frac{1}{2000} \times 1000 = 0.5$ ms

Average, or mean, load voltage, $V_\alpha = \frac{V_0}{100}$

:. Duty cycle, $\alpha = \frac{V_0}{V_s} = \frac{50}{100} = 0.5$

$T_{\text{on}} = 0.5 \times 0.5 = 0.25$ ms ; $T_{\text{off}} = 0.25$ ms.

As chopping period $T = 0.5$ ms is much less than the load time constant $= 6$ ms, the current variation from minimum current $I_1$ to maximum current $I_2$, Fig. 7.5 (d), must be taken as linear. Thus, during $T_{\text{on}}$ period,

$$V_s - V_0 = L \frac{I_2 - I_1}{T_{\text{on}}}$$

or,

$$I_2 - I_1 = \frac{(V_s - V_0)}{L} T_{\text{on}} = \frac{V_0}{V_s} - 1 = \frac{V_s - V_0}{V_s} T_{\text{on}}$$