Ohm's law:

The relationship between voltage (V) and current (I) through resistance (R) in a d.c. circuit was first discovered by German scientist in 1827, George Simon Ohm (1787-1854). This relationship is called Ohm's law and can be stated as:

\[ \frac{V}{I} = \text{constant} = R \]

Where R is the resistance of the conductor between the two points considered.

The ratio of potential difference (V) between the ends of a conductor to the current (I) flowing between them is constant, providing the physical conditions (e.g., temperature etc.) do not change.

Fig. shows if voltage between A & B is `V` volts and current flowing is `I` amperes, then \( \frac{V}{I} \) will be constant and equal to R, the resistance between points A & B.

If the voltage is doubled up, the current will also double up. So that the ratio \( \frac{V}{I} \) remains constant. It may be noted here that, if voltage is measured in volts and current in amperes, the resistance will be in ohms. Ohm's law can also be expressed in three forms viz,

1) \[ I = \frac{V}{R} \]
2) \[ V = IR \]
3) \[ R = \frac{V}{I} \]
**Voltage divider rule**

A series circuit can be considered to be a voltage divider circuit because the potential difference across any one resistor is a fraction of the total voltage applied across the series combination; the fraction being determined by the values of the resistances.

\[
V = I \left( R_1 + R_2 \right) \quad \text{in the total voltage across the series combination of the p.d. across } R_1 \text{ is } V_1 = I \cdot R_1
\]

\[
\therefore \quad \frac{V_1}{V} = \frac{R_1}{R_1 + R_2}
\]

\[
\therefore \quad V_1 = V \cdot \frac{R_1}{R_1 + R_2}
\]

\[
V_2 = V \cdot \frac{R_2}{R_1 + R_2}
\]

Ex

Total resistance \( R = R_1 + R_2 + R_3 = 2 + 4 + 6 = 12 \Omega \)

According to voltage divider rule,

\[
V_1 = V \cdot \frac{R_1}{R_1 + R_2 + R_3} = 24 \cdot \frac{2}{12} = 4 \text{ V}
\]

Similarly,

\[
V_2 = 24 \cdot \frac{4}{12} = 8 \text{ V}
\]

\[
V_3 = 24 \cdot \frac{6}{12} = 12 \text{ V}
\]
A parallel resistor circuit can be considered to current divider circuit because the current through any one resistor is a fraction of the total circuit current, the fraction depending on the value of the resistors.

\[
\begin{align*}
\frac{1}{R_T} &= \frac{1}{R_1} + \frac{1}{R_2} \\
R_T &= \frac{R_1 \cdot R_2}{R_1 + R_2} \quad \text{Parallel resistance} \\
\frac{I_1}{V} &= \frac{V}{R_1} + \frac{V}{R_2} \\
\therefore I_1 &= I \cdot \frac{R_T}{R_1} = I \cdot \frac{R_1 \cdot R_2}{R_1(R_1 + R_2)} \\
\therefore I_1 &= \frac{R_2}{R_1 + R_2} \\
\therefore I_2 &= \frac{R_1}{R_1 + R_2} \\
\end{align*}
\]

Ex.

\[
\begin{align*}
\frac{1}{R_T} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\
R_T &= \frac{R_1 \cdot R_2 \cdot R_3}{R_2 R_3 + R_1 \cdot R_3 + R_1 \cdot R_2} \\
\therefore I_1 &= \frac{V}{R_T} \cdot \frac{R_T}{R_1} = \frac{V \cdot R_2 \cdot R_3}{R_1(R_1 R_2 + R_2 R_3 + R_3 R_1)} \\
\text{Similarly,} \quad I_2 &= I \left( \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \right) \\
I_3 &= I \left( \frac{R_1 R_2}{RR_2 + R_2 R_3 + R_3 R_1} \right)
\end{align*}
\]
How to replace series & parallel combination of resistance by equivalent resistances if How to simplify the network we demonstrate in next part.

Series Resistance:

Consider the combination of three resistances shown in Fig. Recall that in a series circuit the elements are connected end to end and that the same current flows through all of the elements.

By Ohm's law, we can write:

\[ V_1 = R_1 i \]
\[ V_2 = R_2 i \]
\[ V_3 = R_3 i \]

Using KVL, we can write:

\[ V = V_1 + V_2 + V_3 \quad -- \text{(1)} \]

Substitute values for \( V_1, V_2, V_3 \):

\[ V = R_1 i + R_2 i + R_3 i \]

\[ = i (R_1 + R_2 + R_3) \quad -- \text{(2) factoring out the common current } i \]

Now we define the equivalent resistance \( R_{eq} \) to be the sum of the resistances in series:

\[ R_{eq} = R_1 + R_2 + R_3 \quad -- \text{(3)} \]

Substitute in eq. (2):

\[ V = R_{eq} i \quad -- \text{(4)} \]

Thus, we conclude that the three resistance in series can be replaced by the equivalent resistance \( R_{eq} \) with no change in the relationship.
between the voltage \( V \) and current \( i \). If the three resistances are part of a larger circuit, replacing them by a single equivalent resistance would make no changes in the current or voltages in other parts of the circuit.

This analysis can be applied to any number of resistances. For example, two resistances in series can be replaced by a single resistance equal to the sum of the original two.

To summarize, a series combination of resistances has an equivalent resistance equal to the sum of the original resistances.

\[
\text{Parallel Resistances}
\]

[Diagram of parallel resistances]

\[ R_{\text{eq}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \]

Fig. shows three resistances in parallel. In a parallel circuit, the voltage across each current element is the same. Applying Ohm's law, we can write

\[ i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad i_3 = \frac{V}{R_3} \]

The top ends of the resistors are connected to a single node. Thus, we can apply KCL to the top node of the circuit and obtain

\[ i = i_1 + i_2 + i_3 \]

Now put the value of current in above eqn.

\[ i = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \]
factoring out the voltage, we obtain

\[ e = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) i \]

Now, we define the equivalent resistance as

\[ R_{eq} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3} \]

or

\[ R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \]

... \[ i = \frac{1}{R_{eq}} \]

Suppose two resistances are in parallel, then it is equivalent to

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]

or

\[ R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \]

or

\[ R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \]

Hence, we have

Hence, when a number of resistances are connected in parallel, the reciprocal of their total resistance is equal to the sum of reciprocals of individual resistances. The total resistance of a parallel circuit is always less than the smallest of the resistances. If \( k \) resistors, each of resistance \( R_0 \), are connected in parallel, then

\[ R_{eq} = \frac{R_0}{k} \]
Example: Three resistances of 50 Ω, 25 Ω and 5 Ω are connected in series and parallel. Find equivalent resistances.

Solution:
1. \[ R = R_1 + R_2 + R_3 = 50 + 25 + 5 = 80 \, \Omega \]
2. \[ \frac{1}{R} = \frac{1}{50} + \frac{1}{25} + \frac{1}{5} = \frac{1 + 2 + 10}{50} = \frac{13}{50} \]
   \[ R = \frac{50}{13} = 3.846 \, \Omega \]

Example:

Solution:
\[ \frac{1}{R_{CD}} = \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{1}{2} \]
\[ R_{CD} = 2 \, \Omega \]
\[ 3\Omega + 2\Omega = 5\Omega \]

\[ \frac{1}{R_{AB}} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10} = \frac{5+2+1}{10} \]
\[ 10 \Rightarrow R_{AB} = 1.25 \, \Omega \]
\[ \text{Req} = 1.25 \, \Omega \]
Example: Find a single equivalent resistance for the network shown in fig.

\[ R_1 = 10 \, \Omega \quad R_2 = 5 \, \Omega \]

\[ R_2 = 20 \, \Omega \quad \Rightarrow \quad R_4 = 15 \, \Omega \]

\[ \rightarrow \quad R_{eq1} \quad \text{parallel} \quad \Rightarrow \quad R_{eq1} = 10 \, \Omega \]

\[ \frac{1}{R_{eq2}} = \frac{1}{R_2} + \frac{1}{R_{eq1}} \]

\[ R_{eq2} = 10 \, \Omega \]

\[ 10 + 10 = 20 \, \Omega \]

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \]

\[ R_{eq} = \frac{5 \times 3 \times 2}{6} = \frac{1}{6} + \frac{1}{3} + \frac{1}{2} \]

\[ \frac{1}{R_{eq}} = \frac{1 + 2 + 3}{6} = \frac{6}{6} = 1 \quad \Rightarrow \quad R_{eq} = 1 \, \Omega \]

\[ R_{eq} = \frac{1}{3} \quad \rightarrow \quad R = 3 \, \Omega \]
Ex

\[ R_2 = 8 \, \Omega \]

\[ R_{23} = 6 \, \Omega \]

\[ N_{23} = 6 \, \Omega \]

\[ R_{42} = 8 \, \Omega \]

\[ R_{34} = 2 \, \Omega \]

\[ R_{34} = \frac{6 \times 7}{6 + 2} = \frac{6}{3} = 2 \, \Omega \]

\[ R_2 \text{ and } R_{34} \text{ in series} \]

\[ = 8 + 2 = 10 \, \Omega \]

\[ R_1 \text{ and } R_{234} \text{ in parallel} \]

\[ R_{12} = \frac{10}{1 + 1} = 5 \, \Omega \]

\[ \text{Answer} \]
\[ R_{34} = \frac{75}{4} = 18.75 \Omega \]

\[ R_{12} = \frac{1000}{3} = 33.33 \Omega \]

\( R_{12} \) and \( R_{34} \) in series

\[ R_{eq} = R_{12} + R_{34} = 18.75 + 33.33 = 52.08 \Omega \]

\[ R_{eq} \approx 52.1 \Omega \]

\[ R_{12} = 1 + 2 = 3 \text{ k}\Omega \]

\( R_{3} \) and \( R_{12} \) in parallel

\[ R_{eq} = \frac{3}{1 + 1} = \frac{3}{2} = 1.5 \text{ k}\Omega \]
\[ \text{Req} = \frac{1}{\frac{1}{\text{Re}_1} + \frac{1}{\text{Re}_2}} = \frac{1.5 + 1}{\frac{1}{60} + \frac{1}{60}} = \frac{2.5}{\frac{1}{30}} = 24 \Omega \]

\[ \text{Req}_1 + 6 \Omega = 24 + 6 = 30 \Omega = \text{Req}_2 \]

Two 30 \Omega in parallel

\[ \text{Req}_3 = \frac{15}{2} \Omega \]

\[ \text{Req}_4 = \frac{15 + 5}{2} = 10 \Omega \]

\[ \text{Req}_5 = \frac{20}{4} = 5 \Omega \]

Two 25 \Omega in parallel

\[ \text{Req} = \frac{25}{2} = 12.5 \Omega \]

\[ \text{Req} = 5 + 12.5 = 17.5 \Omega \]
Electric Power

The rate at which work is done in an electric circuit is called electric power, i.e.

\[ \text{Electric Power} = \frac{\text{Work done in electric circuit}}{\text{Time}} \]

\[ Q = I \cdot t \]

When voltage applied to a circuit, it causes current to flow through it. Clearly, work in begins done in moving the electrons in the circuit. Thus

\[ V = \text{Voltage across AB} \]
\[ I = \text{current in amp} \]
\[ t = \text{time in sec for which current flows} \]

Total charge that flows in t second is

\[ Q = I \times t \text{ coulomb} \]

\[ V = \frac{\text{work}}{Q} \]

or \[ \text{work} = V \cdot Q = V \cdot I \cdot t \]

Electric power = \[ \frac{\text{work}}{t} = \frac{V \cdot I \cdot t}{t} = V I \text{ joule/sec or Watts} \]

\[ P = V I = I^2 \cdot R = \frac{V^2}{R} \quad \text{-- (by ohms law)} \]
The total work done in an electric circuit is called electrical energy.

\[ \text{Electrical energy} = \text{Electrical power} \times \text{time} \]

\[ = V \cdot I \cdot t \]

\[ \text{or} \quad = I^2 R \cdot t \]

\[ \text{or} \quad = \frac{V^2}{R} \cdot t \]

In practice, electrical energy is measured in kilowatt hour (kWh).

Energy in kWh = Power in kW \times Time in hour.

One kilowatt-hour (kWh) of electrical energy is expended in a circuit if 1 kW (1000 W) of power is supplied for 1 hour.

The electrical bills are made on the basis of total electrical energy consumed by the consumer.

1 unit for change of electricity is 1 kWh.
Gustav Kirchhoff (1824-1887), an eminent German physicist, formulated two fundamental laws in 1847, which are of immense use in the analysis of electric one networks.

1. Current law (KCL), 2. Voltage law (KVL)

Current law or Point law (KCL)

The algebraic sum of currents flowing towards a junction in an electrical circuit is zero, or mathematically, it can be expressed as

at a junction, \( \Sigma I = 0 \)

Consider the simple case of 4 currents carrying conductors meeting at a junction point 0.

If we assume take the signs of current flowing towards 0 as positive, and away from 0 will be negative.

Then according to above law

\[ I_1 + (-I_2) + I_3 + (-I_4) = 0 \]

or \( I_1 + I_3 = I_2 + I_4 \)

or incoming current = outgoing currents.

Hence Kirchhoff's current law can also be stated as under:

The sum of the current flowing towards any junction in an electrical circuit is equal to the sum of current flowing away from the junction.
Voltage Law or Mesh Law:

In any closed circuit or mesh, the algebraic sum of all the electromotive forces (emfs) and the voltage drops is equal to zero. 

\[ \sum \text{EMF} + \sum IR = 0 \]

The validity of Kirchhoff's voltage law can be readily established. If we start in any point in a closed circuit and back to that point after going around the circuit, there is no increase or decrease in potential. This means that the sum of emfs of all & the sources met along the way plus the voltage drops in a resistance must be zero.
Signs convention

of emfs of voltage drops.

A rise in potential should be considered positive while fall in potential should be considered negative.

1. Thus if we go from the positive terminal of battery or source to the negative terminal there is a fall in potential and the e.m.f. should be assigned negative sign. On the other hand, if we go from the negative terminal of battery or source to the positive terminal there is a rise in potential if the e.m.f. should be given +ve sign. (See fig). It may be noted that sign of e.m.f is independent of the direction of current through that branch.

\[ O \quad + \quad 1 \quad \rightarrow \quad R \]

fall in potential fall in potential

\[ O \quad + \quad 1 \quad \rightarrow \quad R \]

rise in potential rise in potential

2. When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in potential because current flows from higher potential to lower potential. Hence this voltage drop should be given -ve sign. If we go against the current flow, there is a rise in potential & the voltage drop should be given +ve sign.
The magnitude of current in any branch circuit can be found by applying KCL. At junction C in above circuit, the incoming cuts to the junction are $I_1$ & $I_2$. Obviously, current in branch CF will be $I_1 + I_2$.

There are 3 closed loops in the above circuit viz. BCFA, CDEFCA & ABCDEFCA. KVL be applied to these loops to get the needed equations.

**ABCDEFCA**

- Start from A, $E_1$ will be given the sign. 
- Age drop in branch CF is $(I_1 + I_2)R_1 < 0$.
- All bear negative sign, we go with current in.
- Hence in a fall in potential. 

Applying KVL to the loop $\nabla$:

$$E_1 - (I_1 + I_2)R_1 = 0$$

$$E_1 = (I_1 + I_2)R_1$$

**CDEFCA**

$$I_2R_2 + (-E_2) + (I_1 + I_2)R_1 = 0$$

$$I_2R_2 + (I_1 + I_2)R_1 = E_2$$

**Loop ABCDEFCA**

$$E_1 + I_2R_2 - E_2 = 0$$

**ABCDEFCA**

$$E_2 - E_1 = I_2R_2$$
Example 3

\[
\begin{align*}
\text{Loop ABCDEA} \\
E_1 + I_1 R_1 - I_2 R_2 + E_2 \\
- I_3 R_3 &= 0
\end{align*}
\]

Method to Solve Circuits by Kirchhoff's Law

1. Assume unknown currents in the given circuit and show their directions by arrows.
2. Choose any closed loop and find the algebraic sum of voltage drops plus the algebraic sum of emfs in that closed loop.
3. Put the algebraic sum of voltage drops plus the algebraic sum of emfs equal to zero.
4. Write equations for many closed loops as the number of unknown quantities.
5. If the value of the assumed current comes out to be negative, it means that actual direction of current is opposite to that of assumed direction.
Example: Calculate the current in 2 Ω resistor in following fig

\[ A \quad I_1 \quad 3.2 \quad C \quad I_2 \quad D \]

\[ 25V \quad I_1+I_2 \quad 4.2 \quad 40V \]

\[ AB \quad \text{CD} \quad EF \quad \text{FA} \]

\[ 35 - 3I_1 - 2(I_1 + I_2) = 0 \]

\[ 35 = 3I_1 + 2I_1 + 2I_2 \]

\[ 5I_1 + 2I_2 = 35 \quad \text{(i)} \]

\[ 35 - 3I_1 + 4I_2 - 40 = 0 \]

\[ 3I_1 - 4I_2 = -5 \quad \text{(ii)} \]

Multiplying eqn \( \text{i} \) \( \times 2 \) & adding to eqn \( \text{ii} \)

\[ 10I_1 + 4I_2 = 70 \quad \text{(i) \times 2} \]

\[ -3I_1 - 4I_2 = -5 \quad \text{(ii)} \]

\[ 13I_1 = 65 \]

\[ I_1 = 5A \]

Put this value in eqn \( \text{i} \)

\[ 5 \times 5 + 2I_2 = 35 \]

\[ 2I_2 = 10 \]

\[ I_2 = 5A \]

Here current through 2Ω resistor is

\[ I_1 + I_2 = 5 + 5 = 10A \]

\[ \therefore \text{Ans. is 10A} \]
Example 8 - Find current in all branches.

Solution:

There are two unknown currents, $I_1$ and $I_2$. Two closed loops will be considered.

**Loop ABCDEFA**
\[
30 - 2I_1 - 10 + 5I_2 = 0
\]
\[-2I_1 + 5I_2 = -20
\]
\[2I_1 - 5I_2 = 20
\]  \(--- (1)

**Loop ECGFE**
\[-5I_2 + 10 - 3(I_1 + I_2) - 5 - 4(I_1 + I_2) = 0
\]
\[-5I_2 - 3I_1 - 3I_2 - 4I_1 - 4I_2 = -5
\]
\[-7I_1 - 12I_2 = -5
\]  \(--- (6)

\[7I_1 + 12I_2 = 5
\]  \(--- (8)

\((1) \times 7\)
\[14I_1 - 35I_2 = 140
\]
\[(6) \times 2\]
\[-14I_1 - 24I_2 = -10
\]

\[-59I_2 = 130
\]
\[I_2 = -\frac{130}{59} = -2.2\ \text{A}
\]

\[
I_2 = 2.2\ \text{A}
\]

From C to F, substituting this value in Eqn (6)

\[2I_1 - 5(-2.2) = 20
\]
\[2I_1 + 11 = 20
\]
\[2I_1 = 9
\]
\[I_1 = \frac{9}{2} = 4.5\ \text{A}
\]

Current in CDEFAB branch
\[I_1 + I_2 = 4.5 + (-2.2) = 2.3\ \text{A}
\]
Example 9: Calculate
1. The magnitude and direction of current in 5Ω resistor B.
2. The resistance between A & C.

Solution: There are 3 unknown quantities viz. $I_1$, $I_2$, & $I_3$. Three closed loops will be considered.

Loop ABDA:

$-I_1 + 5I_3 + 4I_2 = 0$

Case 1:

$-I_1 + 5I_3 = 0$

Case 2:

$I_1 = 4I_2 + 5I_3 = 0$ --- (i)

Loop BCDB:

$-2(I_1 - I_3) + 3(I_2 + I_3) + 5I_2 = 0$

$-2I_1 + 2I_3 + 3I_2 + 3I_3 + 5I_2 = 0$

$-2I_1 + 3I_2 + 10I_3 = 0$

Case 3:

$2I_1 - 3I_2 - 10I_3 = 0$ --- (ii)

Loop FABCEF:

$-I_1 - 2(I_1 - I_3) - 1(I_1 + I_2) + 4 = 0$

$-I_1 - 2I_1 + 2I_3 - I_1 - I_2 = -4$

$-4I_1 - I_2 + 2I_3 = -4$

Case 4:

$4I_1 + I_2 - 2I_3 = 4$ --- (iv)

Step 1:

$x_2 \rightarrow 2I_1 + 4I_3 = 0$

$x_1 - 2I_1 + 2I_2 + 10I_3 = 0$

$x_1 - 2I_1 = 0$

Step 2:

$x_2 \rightarrow 2I_1 - 8I_2 + 10I_3 = 0$

$x_1 - 2I_1 + 3I_2 + 10I_3 = 0$

$-5I_2 + 20I_3 = 0$ --- (iv)
\[
\begin{align*}
\text{(i)} \times 4 & \quad 4I_1 - 16I_2 + 20I_3 = 0 \\
\text{(ii)} & \quad -4I_1 - I_2 + 2I_3 = -4 \\
& \quad -17I_2 + 28I_3 = -4 \\
\text{(iv)} \times 17 & \quad -85I_3 + 340I_3 = 0 \\
\text{(v)} \times 5 & \quad +85I_2 - 110I_3 = 20 \\
\end{align*}
\]

\[230I_3 = 20\]
\[I_3 = \frac{20}{230} = 0.087 \text{ A}\]

Current in \( I_3 \) in \( I_3 \)
\[I_3 = 0.087 \text{ A}\]

Substitute this value in eqn \( \text{(v)} \)
\[5I_2 + 20 \times 0.087 = 0\]
\[5I_2 = 1.74\]
\[I_2 = 0.348 \text{ A}\]

Put value of \( I_2 \) in eqn \( \text{(i)} \)
\[I_1 = 4 \times 0.348 + 5 \times 0.087 = 0\]
\[I_1 = 0.957 \text{ A}\]

Current supplied by battery, \( I = I_1 + I_2 \)
\[I = 0.957 + 0.348 = 1.305 \text{ A}\]

P.D. between \( A \) & \( C \) = EMF of battery - Drop in battery
\[= 4 - 1.305 \times 1\]
\[= 2.695 \text{ V}\]

Resistance between \( A \) & \( C \)
\[\frac{\text{P.D. across AC}}{\text{Battery current(I)}} = \frac{2.695}{1.305} = 2.065 \Omega\]

\[R_{ac} = 2.065 \Omega\]
Source Conversion

A voltage source with a series resistance can be converted into an equivalent current source with parallel resistance. Conversely, a current source with a parallel resistance can be converted into a voltage source.

![Diagram of equivalent circuits]

Fig. 6 shows a voltage source. If we want to convert it into an equivalent current source, then short the terminal A and B as shown in Fig. 6, and current is \( I = \frac{V}{R} \). A current source supplying this current \( I \) and having the same resistance \( R \) connected in parallel with it represent the equivalent source as shown in Fig. 6. Similarly, a current source \( I \) and a parallel resistance \( R \) can be converted into a voltage source of voltage \( V = IR \) and a resistance \( R \) in series with it.

A voltage source—series resistance combination is equivalent to a current source—parallel resistance combination if, and only if, their

1) respective open-circuit voltage are equal, or
2) respective short-circuit current are equal,
3) resistance remains same in both cases.
Convert the voltage source of 10V into an equivalent current source.

**Solution:**

\[ V = 10V \]

Current obtained by putting short across terminals A & B is \[ I = \frac{V}{R} = \frac{10}{5} = 2A \].

**Ex. 2**

Find the equivalent voltage source for the current source in Fig. 3.

**Solution:**

The open circuit voltage across terminals A & B is \[ V_{oc} = \text{drop across } R = \frac{V}{2} = \frac{10}{2} = 5V \].

**Ex. 3**

Use source conversion technique to find the load current in Fig. 3.

**Solution:**

Parallel combination \[ \frac{3 \times 6}{3 + 6} = 2L \]
\[ I = \frac{V}{R} = \frac{6}{3} = 2\, \text{A} \]

Two current sources cannot be combined together because 2 \( \Omega \) resistor present between A and C.

To remove this hurdle, we can convert 2A current source into equivalent with series resistance 4 \( \Omega \) 2 x 2 = 4 \( \Omega \).

Can again be converted into the equivalent current source. Now the two current sources can be combined into a single 4A source.

As the 4A current divided into two equal part at point A because each of the two parallel paths has a resistance 4 \( \Omega \).

Hence \( I_2 = 2\, \text{A} = I_L \)

\( \times 4 \)
Delta-Star Transformation

In solving networks by the application of YV/Ka
one sometimes experience great difficulty due to a
large no. of simultaneous equations that have to be solved.
However, such complicated networks can be simplified
by successively replacing the delta meshes by
equivalent star systems and vice versa.

Suppose we are given three resistances
R12, R23, & R31 connected in delta fashion
between terminals 1, 2 and 3 as shown in Fig. 9. So far
as the respective terminals are concerned, these
three given resistances can be replaced by the
corresponding resistances R1, R2 & R3 connected in star
as shown in Fig. 10.

These two arrangements will be electrically
equivalent if the resistance as measured between any
pair of terminals is the same in both the arrangements.

First take delta connection: Between terminal 1 & 2, there are two parallel paths: one having
a resistance of R12 & the other having a
resistance of (R23 + R31)

The resistance between terminal 1 & 2 is

\[ R_{12} \times \left( \frac{R_{23} + R_{31}}{R_{12} + (R_{23} + R_{31})} \right) \]

Now take star connection: The resistance between the same terminals 1 & 2 is (R1 + R2)

As terminal resistances have to be the
same.
\[ R_1 + R_2 = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \]  \hspace{1cm} (i)

Similarly for 2 + 3, 3 + 1 we get

\[ R_2 + R_3 = \frac{R_{23} \times (R_{12} + R_{31})}{R_{12} + R_{23} + R_{31}} \]  \hspace{1cm} (ii)

\[ R_3 + R_1 = \frac{R_{31} \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \]  \hspace{1cm} (iii)

Now, subtracting (iii) from (i) and adding the result to (ii), we get,

\[ \therefore R_1 = \frac{R_{12} \times R_{23}}{R_{12} + R_{23} + R_{31}} \]

\[ \therefore R_2 = \frac{R_{21} \times R_{12}}{R_{12} + R_{23} + R_{31}} \]

\[ \therefore R_3 = \frac{R_{31} \times R_{23}}{R_{12} + R_{23} + R_{31}} \]

How to remember?

It is seen from above that each numerator is the product of the two sides of the delta which meet at the point in star hence, it should be remembered that: resistance of each arm of the star is given by the product of the resistances of the two delta side that meet at its end divided by the sum of the three delta resistances.
Star-Delta Transformation:

This transformation can be easily done by using equations (1), (3), and (5) given above. Multiplying (1) by (3), (5) with (6) and adding them together and then simplifying them, we get

\[ R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \]

\[ \alpha = R_1 + R_2 + \frac{R_1 R_2}{R_3} \]

\[ R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \]

\[ \alpha = R_2 + R_3 + \frac{R_2 R_3}{R_1} \]

\[ R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \]

\[ \alpha = R_1 + R_3 + \frac{R_3 R_1}{R_2} \]

**How to remember?**

The equivalent delta resistance between any two terminals is given by the sum of the star resistances between those terminals plus the product of these star resistances divided by the third star resistance.
Find the resistance $R_{AB}$.

Solution:

\[ R_{ES} = \frac{4 \times 8}{18} = \frac{32}{18} = \frac{16}{9} \Omega = 1.77 \Omega \]

\[ R_{DE} = \frac{8 \times 6}{18} = \frac{48}{18} = \frac{24}{9} \Omega = 2.66 \Omega \]

\[ R_{DF} = \frac{4 \times 6}{18} = \frac{24}{18} = \frac{12}{9} \Omega = 1.33 \Omega \]

\[ = 6.66 \times 9.33 = 3.88 \Omega \]

\[ = 5.77 \]

\[ = 3.58 \]
Star/Delta & Delta/Star Transformation

\[ R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \]
\[ R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \]
\[ R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \]

\[ R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \]
\[ R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \]
\[ R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \]
Solution
Redraw the circuit

\[ R_{DN} = \frac{10 \times 10}{30} = \frac{10}{3} = 3.33 \Omega \]

Similarly:
\[ R_{CN} = \frac{10}{3} \Omega \]
\[ R_{BN} = \frac{10}{3} \Omega \]

New circuit:

\[ R_{AB} = 10 \Omega \] Answer.
Find the total current I taken from 100 V supply from given network.

Also find current through 8 Ω resistance.

Solution:

Each resistance is calculated: \[ R_{eq} = \frac{6 \times 6}{6 + 6 + 6} = \frac{36}{18} = 2 \, \Omega \]

Current thru 8 Ω resistance:
\[ I = \frac{V}{R} = \frac{100}{10} = 10 \, A \]

\[ I \text{ (ex.2)} = \frac{10}{2} = 5 \, A \]
Problem 4: Calculate current through the 10Ω resistor.

**Solution:**

1. **Series Resistors:**
   
   \[ R_{AN} = R_{CN} = R_{BN} = \frac{12 \times 12}{12 + 12 + 12} = \frac{144}{36} = 4\ \Omega \]
   
   \[ R_{FM} = R_{DM} = R_{EM} = \frac{30 \times 30}{30 + 30 + 30} = \frac{900}{90} = 10\ \Omega \]

2. **Parallel Resistors:**

   \[ R_{EF} = \frac{48 \times 24}{48 + 24} = 16\ \Omega \]

3. **Total Resistance:**

   \[ R_{AP} = \text{Total Resistance} = 4 + 16 + 10 = 30\ \Omega \]

4. **Current Calculation:**

   \[ I = \frac{V}{R} = \frac{180}{30} = 6\ \text{A} \]

   Current through 10Ω resistor is through 24Ω resistors.

   **Applying Current Divider Rule:**

   \[ I_2 = \frac{I 	imes R_1}{R_2 + 24} = \frac{I 	imes R_1}{R_1 + 22} = \frac{6 	imes 48}{48 + 24} = \frac{288}{72} = 4\ \text{A} \]

   Current through 10Ω resistor is 4A.
Calculate the equivalent resistor across battery.

**Setup**

\[ R_{AB} = \frac{1 \times 2}{2 \times 4+1} = \frac{2}{9} = 0.22 \Omega \]

\[ R_{AN} = \frac{1 \times 1}{4} = 0.25 \Omega \]

\[ R_{BK} = \frac{2 \times 1}{4} = 0.5 \Omega \]

**Series**

- Series = 1.25 \Omega
- Series = 1.5 \Omega

**Parallel**

\[ \frac{1.25 \times 0.5}{1.25 + 0.5} = \frac{1.875}{2.75} = 0.68182 \Omega \]

Total resistance \( R_{OB} \):

\[ 0.5 + 0.68182 = 1.18182 \Omega \]

Equivalent resistor = 1.18 \Omega.
Ex (Dec'13, JNTU, 0013) Q.1 (c)

Find the equivalent resistance between terminals A-B in the resistive network of Fig.

**Solution**

\[ R_{AN} = \frac{10 \times 4}{10 + 4 + 4} = \frac{40}{18} = 2.22 \Omega \]

\[ R_{BN} = \frac{4 \times 4}{18} = \frac{16}{18} = 0.888 \Omega \]

\[ R_{CN} = \frac{10 \times 4}{18} = \frac{40}{18} = 2.22 \Omega \]

\[ 15 \text{ Series} = 32.22 \text{ V} \]

\[ 15 \text{ Parallel} = \frac{4.88 \times 22.22}{4.88 + 22.22} = \frac{108.4336}{27.1} = 4.000 \Omega \]

\[ \frac{20 \times 6.22}{20 + 6.22} = 4.74 \Omega \]
\[ (e_1 \parallel k_{r_2} + k_{r_3}) \parallel 2\pi \]
Any arrangement of electrical energy sources, resistances and other circuit elements is called an electrical network. Ohm’s law and Kirchhoff’s laws to solve network problems. Occasions arise when these laws applied to certain networks do not yield quick and easy solution to overcome this difficulty, other network theorems and techniques have been developed which are very useful in analysing both simple and complex circuits. Though the use of these theorem and techniques, it is possible either to simplify the network itself or render the analytical solution easy.

Following network theorems are used to simplify the circuits depend upon the network arrangement:

1) Superposition theorem
2) Thevenin’s theorem
3) Max. power transfer theorem
4) Norton’s theorem etc.
The theorem may be stated as under:

In linear networks containing more than one source of emf, the resultant current in any branch is algebraic sum of the current that would be produced by each emf acting alone, all other sources of emf being replaced meanwhile by their respective internal resistance.

\[ R_T = R_1 + R_p + R_q \quad \text{where} \quad R_p = \frac{R_2 R_3}{R_2 + R_3} \]

Circuit current \( I = \frac{\text{net emf}}{R_T} \)

\[ I = \frac{E_1 - E_2}{R_1 + R_p + R_q} \]

\[ \text{or} \quad I = \frac{E_1}{R_1 + R_p + R_q} - \frac{E_2}{R_1 + R_p + R_q} \]

To show that this statement is true, consider the above circuit. The total circuit resistance \( R_T \) is given by:

Let us now find the circuit current by superposition theorem. Consider that emf short circuit between \( c \) and \( p \).

Current due to \( E_1 \) alone:

\[ I_1 = \frac{E_1}{R_1 + R_p + R_q} \]

Now replace \( E_1 \) with a short circuit between \( A \) and \( E \) so that now \( E_2 \) is acting alone.
Current due to $E_2$ alone,

$$I_2 = -\frac{E_2}{R_1 + R_P + R_4}$$

The minus sign with $I_2$ indicates that the current produced by $E_2$ alone is in a direction opposite to that produced by $E_1$ alone.

2. Circuit current $I =$ Algebraic sum of $I_1$ and $I_2$

$$I = \left( \frac{E_1}{R_1 + R_P + R_4} \right) + \left( -\frac{E_2}{R_1 + R_P + R_4} \right)$$

$$I = \frac{E_1}{R_1 + R_P + R_4} - \frac{E_2}{R_1 + R_P + R_4}$$

Equation 2 is the same with equation 1. This establishes the validity of superposition theorem.

Ex. 1

In fig. a find the different branch currents by superposition theorem.

$$\begin{align*}
&\text{Case 1 by shorting } 40V.
&\text{Total resistance in fig. b across source } \frac{3 + \frac{2 \times 4}{2 + 4}}{2 + 2} = 3 + 1.33 = 4.33 \Omega
\end{align*}$$

Total current $I_1 = \frac{35}{4.33} = 8.08 A$

Current in 4Ω resistor, $I_2' = 8.08 \times \frac{2}{2 + 4} = 2.69 A$

Current in 2Ω resistor, $I_3' = 8.08 \times \frac{4}{2 + 4} = 5.39 A$
Case II

\[ \begin{align*} 
\text{Total resistance across source} & = 4 + \frac{2 \times 2}{2 + 3} = 5.2 \Omega \\
\text{total circuit current} I''_2 & = \frac{40}{5.2} = 7.69 \text{ A} \\
\text{Current in 3 \Omega, } I''_1 & = 7.69 \times \frac{2}{2 + 3} = 3.08 \text{ A} \\
\text{Current in 2 \Omega, } I''_3 & = 7.69 \times \frac{3}{2 + 3} = 4.61 \text{ A} \\
\end{align*} \]

The actual current values of \( I_1, I_2, I_3 \) shown in Fig. @ can be found by algebraically adding the component values:

\[ \begin{align*} 
I_1 = I''_1, & \quad I_1' = I''_1 = 8.08 - 3.08 = 5 \text{ A} \\
I_2 = I''_2, & \quad I_2' = I''_2 = 7.69 - 2.69 = 5 \text{ A} \\
I_3 = I''_3 + I''_3, & \quad I_3' = 5.39 + 4.61 = 10 \text{ A} \\
\end{align*} \]
Ex. 2.

Find the current in each branch of the network.

\[ I_1 \]
\[ \frac{20}{20} = 1 \]
\[ \frac{15.2}{15.2} = 1 \]
\[ \frac{30}{30} = 1 \]
\[ \frac{10}{10} = 1 \]
\[ \frac{20}{20} = 1 \]

\[ I_2 \]
\[ \frac{30}{30} = 1 \]
\[ \frac{10}{10} = 1 \]
\[ \frac{20}{20} = 1 \]

\[ I_3 \]
\[ \frac{15.2}{15.2} = 1 \]
\[ \frac{10}{10} = 1 \]
\[ \frac{20}{20} = 1 \]

**Solution:**

Case I: Shorting 40V & 30V supply.

Total resistance across source = \( 15 + \frac{20 \times 10}{20+10} = 21.67 \Omega \)

\[ I_1' = \frac{20}{21.67} = 0.923 \text{ A} \]

Current in 20Ω, \( I_2' = 0.923 \times \frac{10}{20+10} = 0.307 \text{ A} \)

Current in 10Ω, \( I_3' = 0.923 \times \frac{20}{20+10} = 0.616 \text{ A} \)

Case II: Shorting 20V & 30V supply.

Total resistance across the source = \( 15 + \frac{20 \times 10}{20+15} = 18.57 \Omega \)

\[ I_2'' = \frac{40}{18.57} = 2.15 \text{ A} \]

Current through 20Ω, \( I_2'' = 2.15 \times \frac{15}{20+15} = 0.924 \text{ A} \)

Current through 15Ω, \( I_1'' = 2.15 \times \frac{20}{20+15} = 1.23 \text{ A} \)
**Case III**

**Showing 20Ω & 40Ω Supply**

Total resistance across the source:

\[ R = 20 + \frac{15 \times 10}{15 + 10} = 26 \, \Omega \]

Total circuit current:
\[ I_2''' = \frac{30}{26} = 1.153 \, A \]

Current through 15Ω:
\[ I_1''' = 1.153 \times \frac{10}{10 + 15} = 0.461 \, A \]

Current through 10Ω:
\[ I_3''' = 1.153 \times \frac{15}{10 + 15} = 0.692 \, A \]

The actual values of currents \( I_1, I_2 \) & \( I_3 \) can be found by algebraically adding the component values.

\[ I_1 = I_1' + (-I_1'') + (-I_1''') = 0.923 - 1.22 - 0.461 = -0.768 \, A \]

\[ I_2 = (-I_2') + (-I_2'') + I_2''' = -0.307 - 0.92 + 1.153 = -0.074 \, A \]

\[ I_3 = I_3' + (-I_3'') + I_3''' = 0.616 = -2.15 + 0.692 = -0.842 \, A \]

The negative signs with \( I_1, I_2 \) & \( I_3 \) shows that their actual directions are opposite to the assumed in question's Fig.
Fig 1 shows a network enclosed in a box with two terminals A and B brought out. The network in the box may consist of any number of resistors and emf sources connected in any manner. But according to Thévenin, the entire circuit behind terminals A and B can be replaced by a single source of emf $E_{Th}$ (called Thévenin voltage) in series with a single resistance $R_{Th}$ (called Thévenin resistance) as shown in Fig 2. The value of $E_{Th}$ and $R_{Th}$ are determined as mentioned in Thévenin’s theorem. Once Thévenin’s equivalent circuit is obtained, current through any load resistance $R_L$ connected across AB is given by:

$$I = \frac{E_{Th}}{R_{Th} + R_L}$$

Hence Thévenin’s theorem as applied to d.c. circuit may be stated as under:

Any network having terminals A and B can be replaced by a single source of emf $E_{Th}$ in series with a single resistance $R_{Th}$.

1) The emf $E_{Th}$ is the voltage obtained across terminal A and B with load, if any, removed, i.e., it is the open-circuited voltage between A and B.

2) The resistance $R_{Th}$ is the resistance of the network measured between A and B with load removed and source of emf replaced by their internal resistance.

Solution 8

1) As far as the circuit behind terminals AB is considered, it can be replaced by a single source of emf $E_{Th}$ in series with a single resistance $R_{Th}$ as shown in fig (d).

2) The emf $E_{Th}$ is the voltage across terminals AB with $R_L$ removed. With $R_L$ disconnected, there is no current in $R_2$ and $E_{Th}$ will be the voltage appearing across $R_3$.

$$E_{Th} = \text{voltage across } R_2$$

$$I = \text{current through } R_3 \times \text{Resistance } R_3$$

$$I = \frac{V}{R_1 + R_3} \times R_3$$

To find $R_{Th}$, remove the load $R_L$ and replace the battery by a short-circuit because its internal resistance is assumed zero. Then resistance measured between A & B is equal to $R_{Th}$. Obviously, looking back into the terminal AB, $R_1$ & $R_2$ are in parallel and this parallel combination is in series with $R_3$

$$R_{Th} = \frac{R_2 + \frac{R_1 \times R_3}{R_1 + R_3}}{R_1 + R_3}$$

When load $R_L$ is connected between terminals A and B, then current in $R_L$ is given by

$$I = \frac{E_{Th}}{R_{Th} + R_L}$$
Advantages of Thevenin's equivalent circuit:

1) It greatly simplifies computation when it is necessary to find several values of voltage or current corresponding to several different resistance values in a circuit.

2) It is much easier to simplify the remaining circuitry with its Thevenin equivalent.

---

Ex2. Using Thevenin's theorem, find the current through 8Ω resistor in Fig. Given that battery has an internal resistance of 1Ω.

\[
\begin{align*}
\text{Solution} & \quad \text{Step 1} \\
& \quad \text{Circuit current} = \frac{40}{1+3+6} \\
& \quad = 4 \text{ A} \\
& \quad \text{Voltage across 6Ω resistor,} \\
& \quad E_{th} = 4 \times 6 = 24 \text{ V}
\end{align*}
\]

\[
\text{Step 2}
\]

To find \( R_{th} \), remove the load & replace battery by its internal resistance of 1Ω. The resistance measured between A and B is equal to \( R_{th} \).

\[
R_{th} = 5 + \frac{4 \times 6}{4+6} = 7.4 \Omega
\]

\[
\text{Current in 8Ω} = \frac{E_{th}}{R_{th} + 8} = \frac{24}{7.4 + 8} = 1.56 \text{ A}
\]
Using Thévenin's theorem, find the current in 6Ω resistor in A.

Solution:

9 Battery resistances are not given, it will be assumed that they are zero.

10 To find Eth, remove 6Ω resistor, the voltage between terminal A & B is equal to Eth.

\[
\begin{align*}
4.5 \text{V} & \quad 4.5 - 3 = 1.5 \text{V} \quad \text{Eth} = 3 \text{V} \\
0.167 \text{A} & \quad \text{Total circuit resistance in } 9 \Omega \\
\text{Circuit current} & \quad = \frac{1.5}{9} = 0.167 \text{A}
\end{align*}
\]

The voltage across AB is equal to 4.5 V less drop in 4Ω resistor.

V. Voltage across AB, \( E_{th} = 4.5 - 0.167 \times 4 \)

\[
E_{th} = 3.83 \text{V}
\]

11 To find \( R_{th} \), remove the load ie 6Ω resistor and replace the battery by short. The resistance measured between terminal A & B is equal to \( R_{th} \).

\[
R_{th} = \frac{4 \times 5}{4 + 5} = 2.22 \Omega
\]

\[
E_{th} = 3.83 \text{V}
\]

\[
E_{th} = 3.83 \text{V}
\]

\[
\text{Current in } 6\Omega = \frac{E_{th}}{R_{th} + 6}
\]

\[
= \frac{3.83}{2.22 + 6} = 0.466 \text{ A}
\]
The galvanometer has a resistance of 5.2Ω. Find the current through the galvanometer.

Solution:

\[ V_1 = \frac{10 \times 10}{10 + 15} = \frac{100}{25} = 4 \text{ V} \]

\[ V_2 = \frac{10 \times 12}{12 + 15} = \frac{120}{27} \approx 4.44 \text{ V} \]

Apply KVL in ABDB loop for finding \( V_{Th} \).  

\[ 5 + V_{Th} = 10 \Rightarrow V_{Th} = 5 \]

\[ V_{Th} = 5 - 10 = -5 \text{ V} \]

\[ V_{Th} = 4.28 - 4 = 0.28 \text{ V} \]

\[ R_{Th} = \frac{12.85}{2} \]

\[ R_{Th} = 12.85 \Omega \]
\[ I = \frac{0.28}{12.85 + 5} = 0.01568 \text{A} \approx 15.68 \text{mA} \]

**Example**

\[ 2 \Omega \quad 3 \Omega \quad 4 \Omega \]

\[ 6V \quad 3V \]

**Solution**

\[ \text{Voltage at point a} = \text{Voltage at point b} \]

\[ 6V - V_{\text{mn}} - 3V \]
Fig 1 shows a network enclosed in a box with two terminals A and B brought out. The network in the box may contain any no. of resistors and emf sources connected in any manner. But according to Norton, the entire circuit behind AB can be replaced by a current source In in parallel with a resistance RN as shown in Fig 2. The resistance RN is the same as Thevenin resistance R_N. The value of In is determined as mentioned in Norton's theorem. Once Norton's equivalent circuit is determined, then current through any load R_L connected across AB can be readily obtained.

Norton's theorem as applied to dc circuits may be stated as under:

Any network having two terminals A and B can be replaced by a current source of current output In in parallel with a resistance RN.

i) The output In of the current source is equal to the current that would flow through AB when A & B are short-circuited.

ii) The resistance RN is the resistance of the network measured between A and B with load removed and the source of emf replaced by their internal resistances. The current source is replaced by an open circuit.
The load on the source when AB are short-circuited is given by [Fig 2]

\[ R' = \frac{R_1 + \frac{R_2 R_3}{R_2 + R_3}}{R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3} \]

Source current \[ I' = \frac{V}{R'} = \frac{V (R_2 + R_3)}{R_1 R_2 + R_2 R_3 + R_3 R_1} \]

Short-circuit current \[ I_N = \text{current in } R_2 \]

\[ I_N = I' \times \frac{R_3}{R_2 + R_3} = \frac{V R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \]

To find \( R_N \), remove the load \( R_L \) and replace battery by a short because its internal resistance is assumed zero. (see Fig 3)

\[ R_N = \text{Resistance at terminal AB in Fig 3} \]

\[ R_N = \frac{R_2 + \frac{R_1 R_3}{R_1 + R_3}}{R_1 + R_3} \]

Thus the values of \( I_N \) and \( R_N \) are known.

A Norton equivalent circuit will be as shown in Fig 4.
Ex. 2

Using Norton's theorem, find the current in 8Ω resistor of the network shown in Fig. 1.

Fig. 1

Solution

1) With load 8Ω removed and terminals AB short-circuited in Fig. 2, the current flows through AB is equal to

\[ \text{IN} = \frac{V}{R} = \frac{40}{6.727} = 5.94 \, \text{A} \]

Source current \( I' = \frac{V}{R} = \frac{40}{6.727} = 5.94 \, \text{A} \)

\[ \text{Short circuit current} \, I_n = \frac{I'}{R} = \frac{5.94 \times 6}{11} = 3.24 \, \text{A} \]

Fig. 2

Fig. 3

With load 8Ω removed and battery replaced by a short, the resistance at terminals AB is equal to \( R_N \) as shown in Fig. 3.

\[ R_N = 5 + \frac{4 \times 6}{4 + 6} = 7.4 \, \text{Ω} \]

The Norton's equivalent circuit behind terminals AB is \( I_n = 3.24 \, \text{A} \) in parallel with \( R_N = 7.4 \, \text{Ω} \).

When load 8Ω is connected across AB, Fig. 4. The current source is supplying two resistors, 7.4Ω and 8Ω in parallel.

\[ \text{Current in 8Ω} = \frac{3.24 \times 7.4}{7.4 + 8} = 1.55 \, \text{A} \]
Ex. 3

Using Norton's theorem, find the current in the branch AB, containing 6Ω resistor of the network shown in fig. (1).

Solution

The load of 6Ω is supplied by two sources.

Applying Norton's theorem to the X part in fig. (2):

\[
\text{In} = \frac{4.5}{4} = 1.125 A
\]

\[
\text{RN} = 4 Ω
\]

Similarly, applying Norton's theorem to Y part in fig. (2):

\[
\text{Iny} = \frac{3}{5} = 0.6 A, \quad \text{RN} = 5 Ω
\]

Replacing two current sources by a single current source supplying \(1.125 + 0.6 = 1.725 A\) as shown in fig. (3) & (4), the parallel combination of 4Ω & 5Ω can be replaced by a single resistor equal to \(\frac{4 \times 5}{4+5} = 2.222 Ω\) as shown in fig. (5).

1. Current in 6Ω

\[
= 1.725 \times \frac{2.22}{6+2.22} = 0.466 A
\]
Maximum Power Transfer Theorem

This theorem deals with transfer of maximum power from a source to load and may be stated as:

In any dc circuits, maximum power is transferred from a source to load when the load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all emf sources replaced by their internal resistances.

Fig 1 shows a circuit supplying power to a load $R_L$. The circuit enclosed in the box can be replaced by thevenin's equivalent circuit consisting of thevenin's voltage($V$) in series with thevenin resistance($R_i$) as shown in Fig 2. Clearly, resistance $R_i$ is the resistance measured between terminals $AB$ with $RL$ removed & emf sources & replaced by their internal resistances. According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when $R_L$ is made equal to $R_i$, the thevenin resistance at terminal $AB$.

Consider a voltage source of magnitude $V$ and internal resistance $R_i$ supplying power to load $R_L$ as shown in Fig 3.

Circuit current $I = \frac{V}{R_L + R_i}$
Power delivered to load:

\[ P = I^2R_L = \left( \frac{V}{R_L + R_i} \right)^2 R_L \quad \text{(1)} \]

For a given source, the generated voltage \( V \) and internal resistance \( R_i \) are constant. Therefore, power delivered to the load depends on the value of \( R_L \). In order to find the value of \( R_L \) for which the value of \( P \) is maximum, differentiate eq. (1) w.r.t. \( R_L \) and set the result equal to zero.

Thus,

\[
\frac{dP}{dR_L} = V^2 \left( \frac{R_L + R_i}{(R_L + R_i)^2} - 2 \frac{R_L}{R_L + R_i} \right) = 0
\]

or

\[
(R_L + R_i)^2 - 2R_L(R_L + R_i) = 0
\]

or

\[
(R_L + R_i)(R_L + R_i) - 2R_L(R_L + R_i) = 0
\]

or

\[
R_L^2 + 2R_i R_L + R_i^2 - 2R_L R_i - 2R_L^2 = 0
\]

or

\[
R_L^2 + (R_i - 2R_L) R_L + R_i^2 = 0
\]

\[
R_L = \frac{2R_L - R_i}{2}
\]

Load resistance = Internal resistance of source

Thus, \( R_L = R_i \) for maximum power transfer, the load resistance \( R_L \) should be equal to the internal resistance \( R_i \) of the source. Fig. 4 shows the graph of load power versus load resistance.

We may extend the max. power transfer theorem to a linear circuit rather than a single source by means of Thevenin's theorem as under:

The maximum power is obtained from a linear circuit at a given pair of terminals when terminals are loaded by the Thevenin's equivalent resistance \( (R_{th}) \) of the circuit.

1) The efficiency at maximum power transfer is only 50\% as one-half of the total power generated is dissipated in the internal resistance \( R_i \) of the source.
Efficiency = \frac{\text{Output power}}{\text{Input power}}

\[
= \frac{I^2 R_L}{1^2 (R_L + R_i)} = \frac{R_L}{2 R_L} = \frac{1}{2} = 50\%
\]

ii) Under load conditions of maximum power transfer, the load voltage is one-half of the open circuited voltage at the terminals.

Load voltage = \frac{V}{2}

\[
V_{RL} = \frac{V}{2} - \frac{V}{R_L + R_i} R_L
\]

\[
P_{\text{max}} = \left[ \frac{V}{R_L + R_i} \right]^2 R_L = \left[ \frac{V}{2 R_L} \right]^2 R_L
\]

\[
P_{\text{max}} = \frac{V^2}{4 R_L}
\]

\[
* \frac{dP}{dR_L} = \frac{d}{dR_L} \left( \frac{V}{R_L + R_i} \right)^2 R_L = 0
\]

\[
= V^2 \frac{d}{dR_L} \left[ \frac{R_L}{(R_L + R_i)^2} \right]
\]

\[
\text{we know}\quad d \left( \frac{u}{v} \right) = \frac{v}{u} \frac{du}{dx} - \frac{u}{v} \frac{dv}{dx}
\]

\[
\Rightarrow \frac{d}{dx} u^n = n u^{n-1} \frac{du}{dx}
\]

\[
\frac{dP}{dR_L} = \frac{V^2}{2 R_L} \left[ \frac{(R_L + R_i)^2 x 1 - R_L \cdot 2 (R_L + R_i) \cdot d (R_L + R_i)}{(R_L + R_i)^4} \right]
\]

\[
= \frac{V^2}{2 R_L} \left[ \frac{(R_L + R_i)^2 - 2 R_L (R_L + R_i) x (1 + 0)}{(R_L + R_i)^4} \right]
\]

\[
= \frac{V^2}{2 R_L} \left[ \frac{(R_L + R_i)^2 - 2 R_L (R_L + R_i)}{(R_L + R_i)^4} \right] = 0
\]
Calculate the value of load resistance $R_L$ to which max. power may be transferred from the cir. (fig 1). Determine also the value of max. power.

Solution:

\[ E_{th} = \text{Voltage at AB with } R_L \text{ removed} \]
\[ = \text{Current through } 20 \Omega \times 20 \Omega \]
\[ = \frac{V}{40 + 20} \times 20 = \frac{120}{40 + 20} \times 20 \]
\[ E_{th} = 40 \text{ V} \]

$R_i$ = resistance at AB with $R_L$ removed and 120 V source replaced by a short.

\[ R_i = 60 + \frac{60 \times 20}{40 + 20} \]
\[ = 73.33 \Omega \]

When $R_L$ connected to thevenin equivalent circuit between A & B, circuit shown. It is clear that maximum power will be transferred when:

Thevenin equivalent circuit:

\[ R_L = R_{th} = 73.33 \Omega \]

\[ P_{max} \text{ power to load} = \frac{E_{th}^2}{4R_L} = \frac{40^2}{4 \times 73.33} = 5.45 \text{ Watts} \]
Max power occurs when $R_L = R_2 = 300$. 

\[ R_2 = \frac{800 \times 300}{800 + 300} = \frac{240000}{1100} = 218.18 \Omega \]

The load $R_2$ obtains the maximum power $P_{max} = \frac{1}{2} \times 218.18 \times 218.18 = 150 \text{ W}$.
Current – Voltage relationship in a capacitor.

The charge on a capacitor is given by the expression \( Q = CV \). By differentiating this relation, we get

\[
\frac{dQ}{dt} = \frac{d}{dt} (CV) = C \frac{dv}{dt}
\]

Following important facts can be deduced from above relationship:

i) Since \( Q = CV \), it means the voltage across a capacitor is proportional to charge, not the current.

ii) A capacitor has the ability to store charge and hence to provide a short of memory.

iii) A capacitor can have a voltage across it even when there is no current flowing.

iv) From \( i = C \frac{dv}{dt} \), it is clear that current in the capacitor is present only when on it changes with time. If \( \frac{dv}{dt} = 0 \), i.e., when its voltage is constant or for dc voltage, \( i = 0 \).

Hence, the capacitor behave like an open circuit.

v) From \( i = C \frac{dv}{dt} \), we have \( \frac{dv}{dt} = \frac{i}{C} \). It shows that for a given value of current (charge or discharge) current \( i \), rate of change in voltage is inversely proportional to capacitance. Larger the value of \( C \), slower the rate of change in capacitive voltage. Also, capacitor voltage cannot change instananeously.

vi) The above equation can put as \( dv \equiv \frac{i}{C} dt \)

Integrating the above, we get

\[
\int dv = \frac{1}{C} \int i \, dt \quad \text{or} \quad \Delta v = \frac{1}{C} \int i \, dt
\]
Charging of a capacitor

Fig (a) shows an arrangement by which a capacitor C may be charged through high resistance R from a battery of V volts. The voltage across C can be measured by a suitable voltmeter. When switch S is connected to terminal 'a', C is charged but when it is connected to 'b', C is short circuited through R and is thus discharged. As shown in Fig (b), switch S is shifted to 'a' for charging the capacitor for the battery. The voltage across C does not rise to V instantaneously but built up slowly i.e. exponentially and not linearly. Charging current i<sub>c</sub> is maximum at start i.e. when C is uncharged, then it decreases exponentially and finally ceases when P.D. across capacitor plates becomes equal and opposite to the battery voltage V.
At any instant during charging, let
\[ V_c = \text{p.d. across across } C, \]
\[ i_c = \text{charging current,} \]
\[ q = \text{charge on capacitor plates.} \]

The applied voltage \( V \) is always equal to the sum of:

i) resistive drop \( i_c R \) and
ii) voltage across capacitor \( (V_c) \)

\[ \therefore V = i_c R + V_c \quad \text{-- (i)} \]

Now \[ i_c = \frac{dq}{dt} = \frac{d}{dt} \left( C \cdot V_c \right) \]
\[ = C \cdot \frac{dV_c}{dt} \]

\[ \therefore V = V_c + C \cdot R \frac{dV_c}{dt} \quad \text{-- (ii)} \]

or \[ (V - V_c) = C \cdot R \frac{dV_c}{dt} \]

\[ \therefore \frac{dV_c}{V - V_c} = \frac{dt}{C \cdot R} \]

Integrating both sides, we get
\[ \int \frac{dV_c}{V - V_c} = - \frac{1}{CR} \int dt \]

\[ \therefore \log_e \left( \frac{V - V_c}{V} \right) = \frac{-t}{CR} + \log_e V \quad \text{-- (iii)} \]

Where \( k \) is the constant of integration whose value can be found from initial known conditions.

We know that at the start of charging when \( t = 0 \), \( V_c = 0 \)

Substitute these values in eqn \( \text{iii} \) we get
\[ \log_e V = k \]

Hence eqn \( \text{iii} \) becomes
\[ \log_e \left( \frac{V - V_c}{V} \right) = \frac{-t}{CR} + \log_e V \]
or \[ \log_e \frac{V - V_c}{V} = -\frac{t}{RC} = -\frac{t}{\lambda} \]

where \( \lambda \) is equal to \( CR \) = time constant.

\[ \therefore \frac{V - V_c}{V} = e^{-\frac{t}{\lambda}} \]

or \( V_c = V \left(1 - e^{-\frac{t}{\lambda}} \right) \) \( \text{(iv)} \)

This gives variation with time of voltage across the capacitor plates and is shown in Fig. 8.

\[ \text{Fig. 8} \]

Now \( V_c = \frac{Q}{C} \) \( \therefore V = \frac{Q}{C} \)

Equation \( \text{(iv)} \) becomes

\[ \frac{Q}{C} = \frac{Q}{C} \left(1 - e^{-\frac{t}{\lambda}} \right) \]

\[ \therefore Q = Q \left(1 - e^{-\frac{t}{\lambda}} \right) \] \( \text{(v)} \)

We find that increase of charge, like growth of potential, follows an exponential law in which the steady value is reached after infinite time (Fig. 8), Now \( i_c = \frac{dq}{dt} \)

Differentiating both sides eqn \( \text{(v)} \), we get

\[ \frac{dq}{dt} = i_c = Q \left(\frac{1}{C} \right) \left(1 - e^{-\frac{t}{\lambda}} \right) \]

\[ = Q \left[ 1 + \frac{1}{\lambda} e^{-\frac{t}{\lambda}} \right] \]
\[
\frac{dQ}{dt} = \frac{0}{x} - e^{-t/x}
\]

\[
i_C = \frac{CV}{CR} e^{-t/x}
\]

\[
i_C = \frac{V}{R} e^{-t/x}
\]

or \[i_C = I_0 e^{-t/x}\] (iv)

where \[I_0 = \text{maximum current} = \frac{V}{R}\]

Exponentially rising curves for \(v_C\) and \(q\) are shown in fig 5 and 6 respectively.

Fig 6 shows the curve exponentially decreasing charging current. It should be particularly noted that \(i_C\) decreases in magnitude only but its direction of flow remains the same i.e. positive.

As charging continues, charging current decreases according to equation (iv) as shown in fig 6.

It becomes zero when \(t = \infty\) (though it is almost zero in about 5 times constants). Under steady-state conditions, the circuit appears only as a capacitor which means it acts as an open-circuited.

Similarly, it can be proved that \(v_C\) decreases from its initial maximum value of \(V\) to zero exponentially as given by the relation:

\[v_C = V e^{-t/x}\]
Time Constant

(a) Just at the start of charging, p.d. across capacitor is zero, hence from equation (12) putting
\[ V_c = 0 \], we get
\[ V = CR \frac{dV_c}{dt} \]

\[ \therefore \text{ initial rate o rise of voltage across the} \]

\[ \text{capacitor is} = \left[ \frac{dV_c}{dt} \right]_{t=0} = \frac{V}{CR} \text{ Volt/sec} \]

If this rate of rise were maintained, then time
taken to reach voltage \( V \) would have been

\[ V + V/CR = CR \]. This time is known as

time constant \( \tau \) of the circuit.

Hence, time constant of an R-C circuit is

defined as the time during which voltage

across capacitor would have reached its

maximum value \( V \) had it maintained its

initial rate of rise.

(b) In equation (11) if \( t = \tau \) then

\[ V_c = V (1 - e^{-\frac{t}{\tau}}) \]

\[ = V (1 - e^{-1}) \]

\[ = V [1 - \frac{1}{2.718}] \]

\[ = V [1 - \frac{1}{2.718}] \]

\[ \therefore V_c = 0.632V \]

Hence time constant may be defined as the
time during which capacitor voltage actually
rises to 0.632 of its final steady value.

(c) From equation (11) by putting \( t = \tau \), we get

\[ i_c = I_0 e^{-\frac{t}{\tau}} = I_0 e^{-1} = I_0/2.718 = 0.37 I_0 \]

Hence, constant \( \tau \) of a circuit is also time
during which the charging current falls to 0.37 \% of its

initial maximum value (or fall by 0.632 \% of its initial

value).
Discharging of a capacitor

![Circuit Diagram](image)

In Fig. (a) when switch S is shifted to b, C is discharging through R. It will be seen that the discharging current flows in a direction opposite to that of the charging current as shown in Fig. (b). Hence, if the direction of the charging current is taken positive, then that of the discharging current will be taken as negative. To begin with, the discharge current is maximum but then discharges exponentially till it ceases when capacitor is fully discharged.

Since battery is cut off the circuit, therefore by putting \( V = 0 \) in equation (11)

\[
0 = -R \frac{dv_c}{dt} + i_c
\]

or

\[
v_c = -CR \frac{dv_c}{dt}
\]

\[
\therefore i_c = C \frac{dv_c}{dt}
\]
\[
\frac{dv_c}{dt} = -\frac{t}{C}
\]

or
\[
\frac{dv_c}{v_c} = -\frac{t}{C}
\]

Integrating both sides,
\[
\log_e v_c = -\frac{t}{C} + k
\]

At the start of discharge, when \( t = 0 \), \( v_c = V \).
\[
\log_e V = 0 + k \quad \text{or} \quad \log_e V = k.
\]
Putting this value above, we get
\[
\log_e v_c = -\frac{t}{C} + \log_e V
\]

or
\[
\log_e v_c = -\frac{t}{C}
\]

or
\[
\frac{v_c}{V} = e^{-t/C} \quad \text{or} \quad v_c = V e^{-t/C}
\]

Similarly, \( q = Q e^{-t/C} \) and \( i_c = I_0 e^{-t/C} \).

It can be proved that \( v_R = -V e^{-t/C} \).

The fall of capacitor potential and its discharging current are shown in Fig. 1.

One practical application of the above discharging and the above charging and discharging of a capacitor is found in digital control circuits where square-wave input is applied across an R-C circuit as shown in Fig. 2. The different waveforms of current, \( i_c \), and voltage are shown in Fig. 3, 4, 5, 6.